Today, let's dig into sequences on the GMAT. Let's first understand what a sequence is (from Wikipedia):

A sequence is an ordered list of objects. The number of terms it contains (possibly infinite) is called the length of the sequence. Unlike a set, order matters in a sequence, and exactly the same elements can appear multiple times at different positions in the sequence. Since order matters, (A, B, C) and (B, C, A) are two different sequences. (A series is the sum of the terms of a sequence but we will not deal with series today.)

There are some special sequences e.g. arithmetic progressions and geometric progressions. We will deal with these in subsequent weeks. Today we will look at some generic sequence questions and will learn how to approach them. I will start with a very basic question. Mind you, most sequence questions will be higher level questions since sequence questions look complicated (even though they are very straight forward, believe me!). Let me show you using some questions from external sources:

A note on notation: The first term of a sequence will be denoted by x(1), second term by x(2) and nth term by x(n). (If I want to show multiplication e.g. multiply x by 2, I will show it by writing x*2)

Question 1: In a certain sequence, the term x(n) is given by the formula x(n) = 2*x(n-1) – (1/2)*x(n-2) for all n>= 2. If x(0) = 3 and x(1) = 2, what is the value of x(3)?

(A) 2.5
(B) 3.125
(C) 4
(D) 5
(E) 6.75

Solution: This is a straight forward question event though the formula given is discomforting. Whenever you have a generic formula for the nth term of a sequence, plug in some numbers to see what pattern you get.

x(0) = 3 (given)

x(1) = 2 (given)

If n = 2, x(2) = 2*x(1) – (1/2)*x(0) = 2*2 – (1/2)*3 = 5/2

If n = 3, x(3) = 2*x(2) – (1/2)*x(1) = 2*(5/2) – (1/2)*2 = 4

Answer: C.

I hope you agree that this was very simple. For x(3), you needed x(2). For x(2), you needed x(1) and x(0), both of which you had! So it was a simple matter of quick substitution. Now let's look a teeny bit complicated question

Question 2: The infinite sequence a(1), a(2),… a(n),… is such that a(1) = 4, a(2) = -2, a(3) = 6, a(4) = -1, and a(n) = a(n-4) for n > 4. If T = a(10) + a(11) + a(12) + … a(84) + a(85), what is the value of T?

(A) 119
(B) 120
(C) 121
(D) 126
(E) 133

Solution: We know the first four terms: a(1) = 4, a(2) = -2, a(3) = 6, a(4) = -1

Also it is given that a(n) = a(n-4) i.e. the nth term is equal to the (n-4)th term e.g. 5th term is equal to the 1st term. 6th term is equal to the 2nd term. 7th term is equal to the 3rd term etc.

Hence, the sequence becomes: 4, -2, 6, -1, 4, -2, 6, -1, 4, -2, 6, -1 … (It is always helpful to write down the first few terms of the sequence. It helps you see the pattern.)

The sequence has a cyclicity of 4 i.e. the terms repeat after every 4 terms (go back to the definition of sequence above – it says that the same element can appear multiple times at different positions). Therefore, first to fourth terms will form the first cycle, fifth to eighth terms will form the second cycle, ninth to twelfth terms will form the third cycle and so on…
The sum of each group of 4 terms = 4 – 2 + 6 – 1 = 7

What will be the tenth term, a(10)? A new cycle starts from a(9) so a(9) = 4. Then, a(10) must be -2.

a(10) + a(11) + a(12) is the sum of last three terms of a cycle so this sum must be – 2 + 6 – 1 = 3

a(13) to a(16) is a complete cycle, a(17) to a(20) is another complete cycle and so on… The sum of each of the complete cycles is 7.

How many such complete cycles will there be? The first complete cycle will end at a(16), the second one at a(20), the third one at a(24) etc (i.e. at multiples of 4). The last complete cycle will end at a(84). How many complete cycles do we have here then?

16 = 4*4 and 84 = 4*21 so you start from the fourth multiple to the 21st multiple i.e. you have (21 – 4 + 1) = 18 total cycles. If you are confused about the '+1' here, hang on – I will take it up at the end of this post.

The sum of these 18 cycles will be 7*18 = 126 (I know the multiplication table of 18 as should you!)

We still haven't accounted for a(85), which will be the first term of the next cycle. The first term is 4.

a(10) + a(11) + a(12) + … a(84) + a(85) = 3 + 126 + 4 = 133 = T

or you could just consider this:

You have 18 complete cycles except for the first 3 terms and the last term of the sequence. The last term of the sequence is the first term of a cycle and the first three terms of the sequence are the last three terms of the cycle. So these four terms make one complete cycle. Therefore, instead of 18, you have 19 complete cycles.

T = 7 * 19 = 133

Answer (E)

These were some basic sequences questions. I want to leave you with a sequence question from GMAT prep now. Try and work it out. We will look at its solution next week.

Question 3: For every integer m from 1 to 10 inclusive, the mth term of a certain sequence is given by [(-1)^(m+1)]*[(1/2)^m]. If T is the sum of the first 10 terms in the sequence, then T is:

(A) greater than 2
(B)between 1 and 2
(C) between 0.5 and 1

(D) between 0.25 and 0.5
(E) less than 0.25

Note on the ‘+1’ above: How many numbers are there from 11 to 25, both inclusive? If your answer is 25 – 11 = 14, then you are wrong. When we say 25 – 11, we are saying that we have 25 numbers and we are throwing away 11 of them. But we want to keep the 11 (since we have both inclusive); we want all the numbers starting from 11 and ending at 25. We need to add 1 to the result to ensure that the 11 that we threw out, is retained. Consequently, the number of numbers from 11 to 25, both inclusive is 14 + 1 = 15.